

A holographic computation of the central charges of $d = 4$, $\mathcal{N} = 2$ SCFTs

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ABSTRACT: We use the AdS/CFT correspondence to compute the central charges of the $d = 4$, $\mathcal{N} = 2$ superconformal field theories arising from N D3-branes at singularities in F-theory. These include the conformal theories with E_n global symmetries. We compute the central charges a and c related to the conformal anomaly, and also the central charges k associated to the global symmetry in these theories. All of these are related to the coefficients of Chern-Simons terms in the dual string theory on AdS_5 . Our computation is exact for all values of N , enabling several tests of the dualities recently proposed by Argyres and Seiberg for the E_6 and E_7 theories with $N = 1$.

KEYWORDS: p-branes, AdS-CFT Correspondence, Duality in Gauge Field Theories, Supersymmetry and Duality.

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1. Introduction

Four dimensional $\mathcal{N} = 2$ superconformal field theories (SCFTs) fall into two classes. In one class the gauge coupling is exactly marginal (this is believed to happen in any $\mathcal{N} = 2$ supersymmetric gauge theory with a vanishing one-loop beta function), giving rise to a family of SCFTs which become weakly coupled in some limit (see, e.g., [1]). The second class involves isolated theories with no exactly marginal deformations. These theories cannot be studied perturbatively. In some cases [2–4] one can flow to them from an asymptotically free gauge theory, while in other cases their existence can (so far) only be derived as a low-energy limit of the theory living on some branes in string theory.

A simple example is the low-energy theory living on N D3-branes in F-theory [5–7] compactified on K3, which sit at an F-theory singularity for which the dilaton is constant [8–12]. There are seven types of such singularities, denoted by $H_0, H_1, H_2, D_4, E_6, E_7, E_8$ according to the low-energy gauge theory on the singularity (which is A_n for the H_n -type singularities); this is a global symmetry in the SCFT on the D3-branes (the global symmetry of these theories contains in addition an $SU(2)_L$ symmetry, and an $SU(2)_R \times U(1)_R$ R-symmetry). The singularities may be viewed as groups of 7-branes, which we will call G -type 7-branes for the G -type singularity; they are invariant under the $SL(2, \mathbb{Z})$ duality of type IIB string theory. The SCFT of N D3-branes has an N -dimensional Coulomb branch (which can be visualized as motions of the D3-branes away from the singularity), as well as a non-trivial Higgs branch (equal to the moduli space of N instantons in the group G). In the D_4 case the string coupling is arbitrary, leading to an SCFT with an exactly marginal deformation, which is simply the $USp(2N)$ gauge theory coupled to a hypermultiplet in the anti-symmetric representation and four hypermultiplets

in the fundamental representation [8, 9, 13, 14]. For the other singularities the string coupling is frozen, and the corresponding theory cannot be studied perturbatively. Interest in these theories was recently revived by the work of Argyres and Seiberg [15] which noted that the $N = 1$ E_6 and E_7 theories appeared as subsectors of strong coupling limits of other $\mathcal{N} = 2$ SCFTs with an exactly marginal coupling, providing a Lagrangian formulation for these theories.

$\mathcal{N} = 2$ SCFTs may be characterized by several central charges, which are related to global symmetry “anomalies”. For any $\mathcal{N} = 1$ SCFT, the conformal anomaly involves two central charges a and c , which are related by supersymmetry to the ’t Hooft “anomalies” of the $U(1)_R$ current [16, 17]. The two-point function of the currents of the global symmetry G defines another central charge k_G , which by supersymmetry is related to the “anomalous” 3-point function involving one $U(1)_R$ current and two global symmetry currents. These central charges are independent of exactly marginal deformations, so in theories that have such deformations they may easily be computed in the free field theory limit. On the other hand, it is not clear how to compute them in theories which do not have a weak coupling limit.

In this paper we use the AdS/CFT correspondence [18–20] to compute the central charges of all the $\mathcal{N} = 2$ SCFTs coming from N D3-branes in F-theory. The AdS/CFT dual of these theories was constructed in [21, 22] and it involves type IIB string theory on $AdS_5 \times S^5$, with N units of 5-form flux on the S^5 , and with G -type 7-branes wrapped on an S^3 inside the S^5 , leading to some deficit angle in the circle surrounding them. In these theories the leading order contribution to (some of) the central charges is of order N^2 and comes from the bulk [23, 24], while a correction of order N comes from the effective theory living on the singularity. The central charges may either be computed directly from their definition, or through their supersymmetric relation to anomalous 3-point couplings of currents, which map to coefficients of Chern-Simons terms in the low-energy supergravity on AdS_5 . For a specific central charge of the D_4 theory the order N correction was computed in [25], and here we generalize this computation to all singularity types and to all central charges (though here we do not bother to carefully normalize this contribution, but rather we use the D_4 case to set our normalization). In addition, we provide arguments for the value of the order one contribution to the central charges, leading to an exact formula for the central charges for all values of N and G .

As a first application of our result, we test the dualities proposed in [15]. In that paper the central charge k_G of the $N = 1$ E_6 and E_7 SCFTs was computed assuming the duality, and it was verified that this leads to consistent results for other computations. We compute this central charge holographically, and obtain the same result, thus adding another test for each of the dualities of [15]. In addition, our computations of a and c for these theories provide further tests of these dualities, which are again successful. We hope that our results will be useful for the construction of new dualities involving these theories, and will assist in the classification of $\mathcal{N} = 2$ SCFTs.

We begin in section 2 with a review of central charges in SCFTs, their relation to anomalies, and how they are computed using the AdS/CFT correspondence. In section 3 we review how the $\mathcal{N} = 2$ SCFTs arise from D3-branes near singularities in F-theory.

In section 4 we review the AdS/CFT dual of these SCFTs, and use this to compute their central charges and anomalies. We end in section 5 with a summary of our results, and of how they lead to tests of the dualities of [15].

2. Central charges, anomalies and AdS/CFT

Let us first recall the definitions of central charges in four dimensional SCFTs, and their relation to 't Hooft global symmetry “anomalies” (see, e.g., [17, 26]). In a conformal theory, correlation functions of conserved currents are highly constrained. Two-point functions of currents J_μ^a of a global symmetry group G behave for small x as

$$J_\mu^a(x)J_\nu^b(0) = \frac{3k_G}{4\pi^4}\delta^{ab}\frac{x^2g_{\mu\nu} - 2x_\mu x_\nu}{x^8} + \dots \quad (2.1)$$

We call k_G the central charge of the G -currents, normalizing it (as in [15]) such that a Weyl spinor in the fundamental representation of $SU(2)$ contributes $1/2$ to it.

The central charges contained in correlation functions of the energy-momentum tensor are encoded in its response to a weakly coupled external metric, the conformal anomaly

$$T_\mu^\mu = \frac{c}{16\pi^2}(\text{Weyl})^2 - \frac{a}{16\pi^2}(\text{Euler}), \quad (2.2)$$

where

$$(\text{Weyl})^2 = R_{\mu\nu\rho\sigma}^2 - 2R_{\mu\nu}^2 + \frac{1}{3}R^2, \quad (\text{Euler}) = R_{\mu\nu\rho\sigma}^2 - 4R_{\mu\nu}^2 + R^2. \quad (2.3)$$

a and c can be thought of as measures of the number of degrees of freedom of the theory.

If the theory is superconformal, these central charges are relatively easy to determine, because they are related to 't Hooft anomalies involving the global R-symmetries. For $\mathcal{N} = 1$ SCFTs, a and c are given by [17]

$$a = \frac{3}{32} [3 \text{tr}(R_{\mathcal{N}=1}^3) - \text{tr}(R_{\mathcal{N}=1})], \quad c = \frac{1}{32} [9 \text{tr}(R_{\mathcal{N}=1}^3) - 5 \text{tr}(R_{\mathcal{N}=1})]. \quad (2.4)$$

Here, $\text{tr}(R_{\mathcal{N}=1}^3)$ and $\text{tr}(R_{\mathcal{N}=1})$ denote the strength of the $U(1)_{R,\mathcal{N}=1}^3$ and $U(1)_{R,\mathcal{N}=1}$ -gravity-gravity anomalies, normalized so that they are given by the traces over the labels of Weyl fermions for weakly coupled theories. The R-symmetry of $\mathcal{N} = 2$ SCFTs is $U(1)_R \times SU(2)_R$, and the $U(1)_R$ symmetry of its $\mathcal{N} = 1$ subalgebra is

$$R_{\mathcal{N}=1} = R_{\mathcal{N}=2}/3 + 4I_3/3, \quad (2.5)$$

where I_a ($a = 1, 2, 3$) are the generators of $SU(2)_R$.

Supersymmetry relates the central charge k_G for a flavor symmetry G to the 't Hooft anomaly via the relation

$$k_G \delta^{ab} = -\text{tr}(R_{\mathcal{N}=2} T^a T^b). \quad (2.6)$$

We normalize the generators T^a of G so that they have eigenvalues ± 1 when acting on the adjoint representation. Similarly one can consider the central charges $k_{U(1)_{R,\mathcal{N}=2}}$ and

$k_{\text{SU}(2)_R}$ of the R-symmetries. They are known to be directly proportional to c (they sit in the same supermultiplet with the energy momentum tensor), and are given by

$$k_{\text{U}(1)_{R,\mathcal{N}=2}} = 16c, \quad k_{\text{SU}(2)_R} = 2c. \tag{2.7}$$

Let us now suppose that the SCFT has a gravitational dual defined on AdS_5 . The conserved currents J_a^μ corresponding to the charges Q_a are related to gauge fields A_μ^a living on AdS_5 . The AdS/CFT correspondence equates the bulk action with the partition function of the SCFT with the boundary coupling $\int A_\mu^a J_a^\mu$. This shows a dependence on the gauge of A^a from the cubic 't Hooft anomalies, which is reproduced by the Chern-Simons coupling in the bulk [20]

$$\propto (\text{tr}(Q_a Q_b Q_c)) \int_{AdS_5} \left[A^a \wedge F^b \wedge F^c + \dots \right], \tag{2.8}$$

where \dots denotes the covariantization needed for non-Abelian symmetries. The Chern-Simons interaction is gauge-invariant up to a boundary term, and so the bulk action depends on the gauge of A^a at the boundary. In a similar manner, the U(1)-gravity-gravity anomaly is represented by the mixed gauge-gravity Chern-Simons interaction

$$\propto (\text{tr}(Q_a)) \int_{AdS_5} A^a \wedge \text{tr}(R \wedge R). \tag{2.9}$$

Here, R is the curvature two-form constructed from the five-dimensional metric. Therefore, the calculation of the central charges using the AdS/CFT correspondence reduces to the determination of the Chern-Simons couplings in the dual theory on AdS_5 .

3. F-theory singularities and $\mathcal{N} = 2$ SCFTs

One fruitful way to study supersymmetric field theories in string theory is to realize them as low-energy effective actions on D3-branes probing other branes. To obtain an $\mathcal{N} = 2$ theory, we put D3-branes close to an F-theory singularity, which is characterized by the monodromy of the axiodilaton it produces. To have a superconformal theory, the dilaton gradient close to the singularity must vanish, because this is related to the running coupling constant of the D3-branes.¹ Ordinary (p, q) 7-branes produce logarithmic running of the dilaton, but when one combines several mutually non-local (p, q) 7-branes and puts them at one point, the dilaton can be constant.

The possible singularities with a constant dilaton were classified [6] using the Kodaira classification, and are tabulated in table 1. There, G stands for the low-energy gauge symmetry living on the singularity [10, 27], and n_7 is the number of 7-branes (of different types) used to construct the singularity. 7-branes are heavy, codimension-2 objects, and they produce conical singularities in the transverse space. The total deficit angle is proportional to the number of 7-branes n_7 . For convenience we parametrize the deficit angle by the change in the periodicity of the angular coordinate around the 7-branes,

$$2\pi \rightarrow 2\pi/\Delta. \tag{3.1}$$

¹More precisely, it is related to the effective coupling on the Coulomb branch, but in $\mathcal{N} = 2$ supersymmetric theories this is related to the beta function.

G	H_0	H_1	H_2	D_4	E_6	E_7	E_8
n_7	2	3	4	6	8	9	10
Δ	6/5	4/3	3/2	2	3	4	6

Table 1: Properties of F-theory singularities.

Δ is related to the number of 7-branes by

$$\Delta = \frac{12}{12 - n_7}. \tag{3.2}$$

Note that twelve (p, q) 7-branes produce a deficit angle of 2π , closing up the space, while the D_4 singularity may be viewed as an orientifold (reflecting the transverse space) together with four D7-branes, so it has a deficit angle of π .

Now let us introduce N D3-branes on top of the G -type 7-brane. The field theory realized on the D3-branes has global symmetries coming from the isometry of the system,

$$U(1) \times SO(4) \simeq U(1)_R \times SU(2)_R \times SU(2)_L, \tag{3.3}$$

and also the flavor symmetry G coming from the gauge symmetry of the 7-branes. The theory is of rank N , which means that its Coulomb branch is of complex dimension N . It corresponds to the motion of the N D3-branes in the transverse space to the singularity. If we put the theory at the origin of the Coulomb branch, i.e. if we put all of the D3-branes together on top of the singularity, the theory becomes conformal. One can show that the lowest dimension operator parametrizing the position on the Coulomb branch has dimension Δ , which explains our usage of the symbol Δ for the deficit angle. There is also a Higgs branch emanating from the origin of the Coulomb branch, which will not be relevant for our discussion here.

The only singularity where the value of the dilaton can be arbitrarily tuned is the D_4 singularity, which in the perturbative region may be viewed as a system of four D7-branes on top of an $O7^-$ orientifold plane. The gauge theory on N D3-branes close to it can be found by quantizing the open string, and is an $\mathcal{N} = 2$ $USp(2N)$ gauge theory with eight half-hypermultiplets in the fundamental representation $\mathbf{2N}$, and two half-hypermultiplets in the antisymmetric tensor $\mathbf{N(2N - 1)}$. The half-hypermultiplets in $\mathbf{2N}$ transform as the fundamental of the $D_4 = SO(8)$ flavor symmetry, and the half-hypermultiplets in $\mathbf{N(2N - 1)}$ as the fundamental of $SU(2)_L$. The trace part of the antisymmetric tensor corresponds to the overall motion of the D3-branes parallel to the 7-brane, which is completely decoupled from the rest of the theory.

The monodromy around other types of 7-branes fixes the dilaton, so these systems are inherently strongly coupled. The theories on one D3-brane near the 7-brane of type H_n correspond to the $\mathcal{N} = 2$ SCFTs found in [2, 3], and can be realized field theoretically by tuning the masses and the vacuum expectation value of an $SU(2)$ gauge theory with $n + 1$ massive fundamental flavors. The superconformal R-symmetry is an accidental symmetry at the infrared fixed point, so we do not have any effective means of calculating the 't Hooft anomalies in the language of the four dimensional field theory. For the 7-brane of

type E_n , the corresponding SCFTs on a single D3-brane were first discussed in [9, 10] and were studied shortly thereafter in detail by [11, 12]. A purely field-theoretical construction of these theories was not known until quite recently when the paper [15] appeared, which motivated us to re-analyze these theories.

4. Anomalies and central charges from the AdS description

Let us determine the central charges of the SCFTs described in the previous section via the AdS/CFT correspondence. The AdS/CFT dual of these theories was constructed in [21, 22], by taking the near-horizon limit of N D3-branes sitting on the 7-brane. This gives type IIB string theory on $AdS_5 \times X_\Delta$, with N units of F_5 flux. Here X_Δ is the round 5-sphere

$$\{|x|^2 + |y|^2 + |z|^2 = \text{constant}\} \subset \mathbb{C}^3, \tag{4.1}$$

with the phase of z restricted to $[0, 2\pi/\Delta]$ and periodically identified. The 7-brane of type G wraps the locus $z = 0$, which is a round S^3 . Massless gauge fields on AdS_5 arise both from the isometries of X_Δ , and from the gauge fields on the 7-brane. Our objective is to find their Chern-Simons interactions, or equivalently, the conformal anomaly coefficients a and c and the central charges k_G and k_L of the flavor symmetries G and $SU(2)_L$, respectively. We discuss the $\mathcal{O}(N^2)$, $\mathcal{O}(N)$ and $\mathcal{O}(1)$ contributions in this order, and denote the $\mathcal{O}(N^2)$ contribution to a by $a^{(2)}$, etc.

4.1 $\mathcal{O}(N^2)$ contributions

The $\mathcal{O}(N^2)$ contributions to the anomalies come only from the gravity in the bulk, since the action of the 7-brane is of order N [25]. The conformal anomalies a and c were determined by [23] to be

$$a = c = \frac{N^2 \pi^3}{4 \text{vol}(X_5)} \tag{4.2}$$

for general Einstein manifolds X_5 , where $\text{vol}(X_5)$ is the volume of X_5 , normalized to have a unit radius of curvature.

The scaling with N and with the volume of all terms in the classical bulk action on AdS_5 is easily found on dimensional grounds [24]. Let us use the coordinate system on $AdS_5 \times X_\Delta$ which is independent of the radius of curvature R_{AdS} of AdS_5 , and put the factor R_{AdS}^2 in the metric. Then, the ten dimensional Lagrangian density scales in Planck units as R_{AdS}^8 . We have N units of 5-form flux on X_5 , so N is related to this by $N \propto R_{\text{AdS}}^4 \text{vol}(X_5)$. Thus, the Lagrangian density of the ten-dimensional bulk scales as $(N/\text{vol}(X_5))^2$. Integrating it over X_5 gives the five-dimensional Lagrangian density scaling as $N^2/\text{vol}(X_5)$, which is reflected in the scaling in (4.2). In our case the volume of X_Δ is that of a 5-sphere divided by Δ , so we get

$$a^{(2)} = c^{(2)} = \frac{N^2 \Delta}{4} \tag{4.3}$$

to this order.

As mentioned above, the same central charges correspond to the Chern-Simons interactions among the R-symmetry gauge fields, so they can also be derived by decomposing the $SU(4)_R^3$ Chern-Simons interactions of type IIB supergravity on $AdS_5 \times S^5$, because X_Δ is locally the same as S^5 . This decomposition includes also the $U(1)_R SU(2)_L^2$ Chern-Simons term, whose strength has a fixed ratio relative to the strength of the $(U(1)_{R,N=1})^3$ Chern-Simons term (which is also in the decomposition). The latter R-symmetry anomaly is fixed by the superconformal algebra (2.4). Thus, we get the $\mathcal{O}(N^2)$ contribution to k_L

$$k_L^{(2)} = \frac{N^2 \Delta}{2}, \tag{4.4}$$

where we used the relation (2.6) between the central charge and the anomaly. Finally, there is obviously no bulk contribution to the Chern-Simons term of the G flavor symmetry, which lives only on the 7-brane. In the D_4 case it is easy to compute all of these central charges in the free field theory limit, leading to the same results (at leading order in $1/N$).

4.2 $\mathcal{O}(N)$ contributions

In the bulk there are no $\mathcal{O}(N)$ contributions to the central charges and anomalies, since the one-loop corrections in the bulk are $\mathcal{O}(1)$. Thus, the $\mathcal{O}(N)$ contributions to the anomalies come purely from the Chern-Simons interactions on the 7-brane at the singularity, which include terms of the form $C_4 \wedge \text{tr}(R \wedge R)$ and $C_4 \wedge \text{tr}(F \wedge F)$. The dimensional reduction of these terms gives rise to five-dimensional Chern-Simons interactions, since the five dimensional gauge fields involving the isometries include [28], in addition to the ten dimensional metric, a contribution of the form $C_4 \sim A_R \wedge \omega$ where A_R is the $U(1)_R$ gauge potential on AdS_5 and ω is the volume form of the 3-cycle wrapped by the singularity.

To determine the terms on the 7-brane, let us first recall that the 7-brane of type D_4 can be realized perturbatively as 4 D7-branes put on top of the $O7^-$ -plane. The Chern-Simons coupling on the worldvolume to the four-form field C_4 was determined in [25]. The other types of 7-branes have the dilaton pinned down to the strong coupling region, but their Chern-Simons terms are related to the anomaly inflow and can still be reliably determined. Each constituent (p, q) 7-brane carries the same coupling to $C_4 \wedge \text{tr}(R \wedge R)$ (recall that all (p, q) 7-branes are related by the $SL(2, \mathbb{Z})$ duality symmetry which leaves C_4 invariant), so when we bring several 7-branes together the strength of that term is proportional to n_7 . As for the coupling $C_4 \wedge \text{tr}(F \wedge F)$, the gauge symmetries on the worldvolume for various types of 7-branes are related by the removal of (p, q) 7-branes one by one which enables flows between the different theories, with a natural embedding of the (simply laced) symmetries

$$A_1 \subset A_2 \subset D_4 \subset E_6 \subset E_7 \subset E_8. \tag{4.5}$$

Therefore, the strength of the coupling does not depend on the type of the 7-brane, as long as we use the same normalization of the root vectors. We conclude that the Chern-Simons terms on the 7-brane worldvolume are of the form

$$A n_7 \int C_4 \wedge [\text{tr}(R_T \wedge R_T) - \text{tr}(R_N \wedge R_N)] + B \int C_4 \wedge \text{tr}(F \wedge F), \tag{4.6}$$

with constants A and B independent of the type of the 7-brane. Here $R_{T,N}$ are the curvature of the tangent bundle and the normal bundle of the 7-brane, and we normalize the trace of the flavor symmetry so that $\text{tr}(T^a T^b) = \delta^{ab}/2$ independent of the group.

These terms reduce to various Chern-Simons interactions in five dimensions after the integral over the S^3 wrapped by the 7-brane. The $C_4 \wedge \text{tr}(F \wedge F)$ term gives rise to the $U(1)_R G^2$ Chern-Simons term on AdS_5 , while the $C_4 \wedge \text{tr}(R_T \wedge R_T)$ term in (4.6) produces $U(1)_R^3$ and $U(1)_R$ -gravity-gravity Chern-Simons terms. Finally, the $C_4 \wedge \text{tr}(R_N \wedge R_N)$ term gives $U(1)_R SU(2)_R^2$ and $U(1)_R SU(2)_L^2$ Chern-Simons interactions. Together, the $C_4 \wedge \text{tr}(R \wedge R)$ terms contribute to the anomalies appearing in (2.4), and thus to a and c .

Again, we can easily determine the scaling with N and with the volume of X_Δ of the terms in the five dimensional action arising from the integration of (4.6) on the 3-sphere and involving the $U(1)_R$ field A_R . On dimensional grounds, the full low-energy 7-brane Lagrangian density is proportional to $R_{\text{AdS}}^4 \sim N/\text{vol}(X_5)$. On the other hand, the volume of the 3-sphere which the 7-brane wraps is independent of the deficit angle Δ . Thus, the coefficients of the five dimensional terms we obtain from (4.6) scale as $N/\text{vol}(X_5) \propto N\Delta$.

Therefore, we find that the $\mathcal{O}(N)$ correction to the various anomalies scales as

$$k_G^{(1)} \propto N\Delta, \quad k_L^{(1)}, a^{(1)}, c^{(1)} \propto Nn_7\Delta. \tag{4.7}$$

The coefficients can be fixed by a careful computation, but instead we will fix them by comparing them to the perturbative case D_4 , for which we can easily find

$$k_G^{(1)} = 4N, \quad k_L^{(1)} = -N/2, \quad a^{(1)} = N/2, \quad c^{(1)} = 3N/4, \tag{4.8}$$

from the spectrum of the gauge theory. As tabulated in table 1, $\Delta = 2$ and $n_7 = 6$ for D_4 . Thus, we conclude that for all the theories

$$k_G^{(1)} = 2N\Delta, \quad k_L^{(1)} = -\frac{Nn_7\Delta}{24}, \quad a^{(1)} = \frac{Nn_7\Delta}{24}, \quad c^{(1)} = \frac{Nn_7\Delta}{16}. \tag{4.9}$$

4.3 $\mathcal{O}(1)$ contributions

Relatively little is known about the $\mathcal{O}(1)$ corrections to the anomaly coefficients. For the prototypical duality between the $\mathcal{N} = 4$ $SU(N)$ super Yang-Mills theory and the $AdS_5 \times S^5$ background, the $\mathcal{O}(1)$ correction accounts for the difference between the gauge group $U(N)$ and $SU(N)$. On the stack of N D3-branes in a flat space, we naturally have a $U(N)$ gauge symmetry, where the $U(1)$ part describes the center-of-mass motion of the D3-branes in the transverse \mathbb{R}^6 . This overall motion decouples from the rest of the dynamics in the near-horizon limit, and it is not the part of the theory dual to type IIB string theory on $AdS_5 \times S^5$.

In type IIB string theory on $AdS_5 \times S^5$, the Kaluza-Klein reduction from ten dimensions gives the coefficient N^2 for the $SU(4)_R^3$ Chern-Simons term, and the problem is how to account for the extra (-1) in the coefficient of the Chern-Simons interaction. It was argued in [29] that the one-loop integral of the fermionic Kaluza-Klein towers reproduces this contribution. The reasoning goes roughly as follows: almost all of the states in the fermionic tower come in pairs of an $SU(4)_R$ representation and its conjugate so that they

do not contribute to the Chern-Simons term, but one chiral fermion in the $\mathbf{4}$ of $SU(4)_R$ is missing in the spectrum. This fermion is in the so-called “doubleton” representation of $SO(4, 2)$, which naively appears in the Kaluza-Klein reduction, but in fact corresponds to pure gauge modes. The extra (-1) in the Chern-Simons coefficient comes from the careful one-loop integration of this fermion, which can be identified with the fermion in the $\mathcal{N} = 4$ vector multiplet corresponding to the overall motion of the D3-branes. Thus, the one-loop contribution gives precisely minus the contribution of a single $\mathcal{N} = 4$ vector multiplet.

Another argument giving the same answer is the following. In the system of D3-branes before taking any near-horizon limit, the global anomaly must be canceled by some anomaly inflow (as discussed e.g. in [30, 31] for the case of M5-branes). However, all inflow terms involve the fields generated by the D3-branes, so they are proportional to some positive power of N .² Thus, there cannot be any terms of $\mathcal{O}(1)$ (or higher orders in $1/N$) in the anomaly of the D3-branes. This anomaly includes both the non-trivial SCFT of the D3-branes and the decoupled vector multiplet; thus, the $\mathcal{O}(1)$ terms in the non-trivial SCFT are precisely minus the contribution of a single $\mathcal{N} = 4$ vector multiplet, as above.

We can easily generalize both types of arguments to our setup which is $\mathcal{N} = 2$ supersymmetric. Here, the degrees of freedom which decouple in the near-horizon limit correspond to the overall motion parallel to the 7-brane, which is described by a free hypermultiplet which transforms in the $\mathbf{2}$ of $SU(2)_L$, and is neutral under the flavor symmetry. Again, this free hypermultiplet is in a one-to-one correspondence with the “doubleton” fields on $AdS_5 \times X_\Delta$. Thus, the arguments above suggest that the $\mathcal{O}(1)$ contributions in this case should be precisely minus those of this free hypermultiplet, namely

$$k_G^{(0)} = 0, \quad k_L^{(0)} = -1/2, \quad a^{(0)} = -1/24, \quad c^{(0)} = -1/12, \quad (4.10)$$

independently of the type of the 7-brane. This precisely agrees with the known result in the D_4 case, and we will see more evidence in the next section that this procedure is correct.

5. Summary of results and tests of Argyres-Seiberg dualities

Combining the results obtained so far, we obtain our final equations

$$\begin{aligned} k_G &= 2N\Delta, \\ k_L &= \frac{1}{2}N^2\Delta - \frac{1}{2}N(\Delta - 1) - \frac{1}{2}, \\ a &= \frac{1}{4}N^2\Delta + \frac{1}{2}N(\Delta - 1) - \frac{1}{24}, \\ c &= \frac{1}{4}N^2\Delta + \frac{3}{4}N(\Delta - 1) - \frac{1}{12} \end{aligned} \quad (5.1)$$

for the central charges. Here, we used the relation (3.2) which relates the deficit angle and the number of 7-branes to rewrite $n_7\Delta = 12(\Delta - 1)$.

²In other words, the inflow vanishes for $N = 0$. We assume that there are no bulk fields localized in the same place that the D3-branes are localized; this argument can fail when the D3-branes sit at a fixed point of some orbifold.

G	H_0	H_1	H_2	D_4	E_6	E_7	E_8
k_G	$\frac{12}{5}$	$\frac{8}{3}$	3	4	6	8	12
a	$\frac{43}{120}$	$\frac{11}{24}$	$\frac{7}{12}$	$\frac{23}{24}$	$\frac{41}{24}$	$\frac{59}{24}$	$\frac{95}{24}$
c	$\frac{11}{30}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{7}{6}$	$\frac{13}{6}$	$\frac{19}{6}$	$\frac{31}{6}$

Table 2: Central charges of rank one SCFTs. The D_4 case was used as an input to set the normalizations. We found agreement with the results of [15] for the $E_{6,7}$ cases.

Based on the arguments above we believe that these formulas are exact, so we can use them even in the case of $N = 1$. The first thing we notice is that k_L automatically vanishes if $N = 1$. This provides us with a test of our procedure in the last section, because the motion of a single D3-brane along the 7-brane is always described purely by the free hypermultiplet, and hence the rank one SCFTs do not carry the extra flavor symmetry $SU(2)_L$. The rest of the central charges, k_G , a and c for the rank one SCFTs are tabulated in table 2. Note that in the H_0 case there is no flavor symmetry G , so the value of k_G which we wrote (using the formulas above) is not really meaningful.

Let us next compare our results to the recent field-theoretical analysis in [15]. There, the authors analyzed rank two $\mathcal{N} = 2$ supersymmetric gauge theories whose beta function is perturbatively zero. The gauge coupling constant of such theories is exactly marginal, so one can go to the strong coupling limit while preserving the superconformal symmetry. They used the Seiberg-Witten curves encoding the effective couplings on the Coulomb branch to study two cases, and found that a specific strong coupling limit of these theories is given by the isolated rank one SCFTs of type $E_{6,7}$, weakly coupled to an $SU(2)$ theory (with additional fundamental half-hypermultiplets in one case). This is a new type of S-duality, and it is also the first construction of isolated SCFTs with exceptional symmetry which uses purely four-dimensional field theory. Furthermore, the R-symmetry of the setup is manifest throughout the procedure, so it is easy to calculate the conformal anomalies a and c .

The first case is the duality

$$SU(3) \text{ with } 6 \times (\mathbf{3} + \bar{\mathbf{3}}) \longleftrightarrow SU(2) \text{ with } 2 \times \mathbf{2} \text{ and SCFT}_{E_6}, \tag{5.2}$$

where $SU(2) \subset E_6$ is gauged. The consistency of this duality requires [15] the E_6 central charge to be $k = 6$, and this matches precisely with our calculation above using AdS/CFT. If the duality is correct, we can compute a and c of the E_6 SCFT by calculating those of the $SU(3)$ gauge theory and subtracting those of the $SU(2)$ gauge theory (including the hypermultiplet contributions) on the right hand side.³ The results are

$$a = \frac{29}{12} - \frac{17}{24} = \frac{41}{24}, \quad c = \frac{17}{6} - \frac{2}{3} = \frac{13}{6}, \tag{5.3}$$

³In [15] the authors calculated in the same way the central charge $k_{U(1)_R}$ of the $U(1)_R$ current, which is proportional to c by the relation (2.7).

which also perfectly match with our results above.

The second case is

$$\mathrm{USp}(4) \text{ with } 12 \times 4 \longleftrightarrow \mathrm{SU}(2) \text{ with SCFT}_{E_7}. \quad (5.4)$$

The consistency of the duality in this case requires the flavor current central charge to be $k = 8$ [15]. a and c can be obtained just as before, with the results

$$a = \frac{37}{12} - \frac{5}{8} = \frac{59}{24}, \quad c = \frac{11}{3} - \frac{1}{2} = \frac{19}{6}. \quad (5.5)$$

These also completely agree with our findings, tabulated in table 2.

These non-trivial agreements strongly suggest that the new S-duality found in [15] and our calculation in this paper are both consistent and correct. In table 2, the entry for D_4 was used as an input (for normalizing the $\mathcal{O}(N)$ terms), and we compared successfully the entries for E_6 and E_7 against the results obtained in [15]. The other entries are our prediction. It would be extremely interesting to calculate k , a and c of other isolated SCFTs using some purely field theoretical framework (such as new S-dualities), and to compare them to our findings. Our results may be useful for pursuing that direction. For example, the fact that c of the rank one E_8 theory is larger than c of any rank two perturbative SCFT strongly suggests that we need to look for SCFTs of rank more than two in order to find a dual for a theory including the rank one E_8 theory. This is also suggested by the fact that since this theory has $k_G = 12$, we cannot gauge an $\mathrm{SU}(2)$ subgroup of E_8 , since the resulting $\mathrm{SU}(2)$ theory would not be asymptotically free.

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